

Thermodynamic Lattice Study for Preconformal Dynamics in Strongly Flavored Gauge Theory

Kohtaroh Miura

INFN - Laboratori Nazionali di Frascati Via Enrico Fermi, n.40 Frascati Rome Italy

E-mail: Kohtaroh.Miura@lnf.infn.it

Abstract. By using the lattice Monte-Carlo simulation, we investigate the finite temperature chiral phase transition at color SU(3) gauge theories with various species of fundamental fermions, and discuss the signal of the (pre-)conformality at large N_f (num. of flavors.) via their comparisons. With increasing N_f , we confirm stronger fermion screenings resulting from a larger fermion multiplicity. We investigate a finite T step-scaling which is attributed to the uniqueness of the critical temperature (T_c) at each N_f , then the vanishing step-scaling signals the emergence of the conformality around $N_f^* \sim 10 - 12$. Further, motivated by the recent functional renormalization group analyses, we examine the N_f dependence of T_c , whose vanishing behavior indicates the conformality at $N_f^* \sim 9 - 10$.

1. Introduction

Conformal invariance is anticipated to emerge in asymptotically free non-Abelian gauge theories when the fermion species (flavor N_f) exceeds a critical value $N_f = N_f^*$ [1, 2]. The approach to the conformality from below is in principle associated with a pre-conformal (walking) behavior of the running coupling, which has been advocated as a basis for strongly interacting mechanisms of electroweak symmetry breaking [3].

Recent lattice studies [4] focused on the computation of N_f^* and the analysis of the conformal window itself, either with fundamental fermions or other representations. Among the many interesting results with fundamental fermions, we single out the observation that the color SU(3) gauge theory with $N_f = 8$ is still in the hadronic phase [5, 6], while $N_f = 12$ seems to be close to the critical number of flavors, with some groups favoring conformality [6–9], and others chiral symmetry breaking [10].

In order to attack the walking and the conformal dynamics, it is more informative beyond a fixed N_f to investigate the vanishing or reducing chiral dynamics with increasing varying N_f . To this end, we investigate the N_f dependences of the chiral phase transition at finite temperature (T) based on our recent works [5, 11]: The vanishing (reducing) finite T step-scaling which attributes to the uniqueness of T_c at each N_f signals the emergence of the (pre-)conformality. Further, motivated by the recent functional renormalization group (FRG) studies [12], we examine the N_f dependence of T_c , whose vanishing (decreasing) behavior indicates the (pre-)conformality. This *thermodynamic* lattice study for the large N_f non-Abelian gauge theory has played a crucial role to extract a notion of more strongly interacting non-Abelian plasma [13], and it is expected to provide a new connection between the lattice and the Gauge/Gravity duality [14].

2. Simulation setups

Simulations have been performed by utilizing the publicly available MILC code [15]. We use an improved version of the staggered action, the Asqtad action, with a one-loop Symanzik [16] and tadpole [17] improved gauge action. The tadpole factor u_0 is determined by performing zero temperature simulations on the 12^4 lattice, and used as an input for finite temperature simulations. To generate configurations with mass degenerate dynamical flavors, we have used the rational hybrid Monte Carlo algorithm (RHMC) [18].

Our observables are the chiral condensate and the Polykov loop

$$a^3 \langle \bar{\psi} \psi \rangle = \frac{N_f}{4N_s^3 N_t} \langle \text{Tr}[M^{-1}] \rangle, \quad L = \frac{1}{N_c N_s^3} \sum_{\mathbf{x}} \text{Re} \left\langle \text{tr}_c \prod_{t=1}^{N_t} U_{4,t\mathbf{x}} \right\rangle, \quad (1)$$

where N_s (N_t) represents the number of lattice sites in the spatial (temporal) direction and $U_{4,t\mathbf{x}}$ is the temporal link variable. The other important observable is the ratio of a scalar and a pseudo-scalar susceptibility [19],

$$R_\pi \equiv \frac{\chi_\sigma}{\chi_\pi} = \frac{\partial \langle \bar{\psi} \psi \rangle / \partial m}{\langle \bar{\psi} \psi \rangle / m} = \frac{\chi_{\text{conn}} + \chi_{\text{disc}}}{\langle \bar{\psi} \psi \rangle / m}, \quad (2)$$

where $a^2 \chi_{\text{conn}} = -N_f \langle \text{Tr}[(MM)^{-1}] \rangle / (4N_s^3 N_t)$ and $a^2 \chi_{\text{disc}} = N_f^2 [\langle \text{Tr}[M^{-1}]^2 \rangle - \langle \text{Tr} M^{-1} \rangle^2] / (16N_s^3 N_t)$. Here, $R_\pi \sim \mathcal{O}(1)$ indicates the scalar and pseudo-scalar degeneracy attributed to the approximate chiral restoration, while $R_\pi \ll 1$ indicates the chiral symmetry breaking [5, 19].

3. Results

We evaluate the thermalized ensemble averages of the chiral condensate (PBP) and Polyakov loop (PLOOP) for various lattice couplings β_L , lattice sizes with the finite T set up $N_s \gg N_t$, and the number of flavors N_f . All results have been obtained by using a single value for a lattice bare fermion mass $am = 0.02$. Then we locate the lattice bare coupling β_L^c associated with the chiral crossover which is signaled by the drastic decrease (increase) of PBP (PLOOP) as a function of β_L . In practice, the ratio of the scalar and pseudo-scalar susceptibility R_π gives a stronger signal of the chiral crossover, owing to its renormalization invariant property. In table 1, we summarize the obtained critical lattice couplings as a function of (N_f, N_t) . We have confirmed the (approximate) asymptotic scaling for the normalized critical temperature T_c/Λ_L varying N_t at each N_f , where Λ_L is so-called lattice Lambda. This indicates that our β_L^c have been determined near to the continuum limit [20].

Table 1. Summary of the (pseudo) critical lattice couplings β_L^c for the theories with $N_f = 0, 4, 6, 8$, $am = 0.02$ and varying $N_t = 4, 6, 8, 12$. The result has partially been extracted from our recent paper [5, 11]. All results are obtained by using the action with the same level improvements.

$N_f \backslash N_t$	4	6	8	12
0	7.35 ± 0.1	7.88 ± 0.05	8.20 ± 0.1	—
4	5.60 ± 0.1	5.89 ± 0.05	6.10 ± 0.1	—
6	4.65 ± 0.05	5.05 ± 0.05	5.2 ± 0.05	5.55 ± 0.1
8	—	4.1125 ± 0.0125	4.20 ± 0.1	4.34 ± 0.04

We shall now discuss the emergence of the (pre-)conformality by using our critical lattice $g_L^c = \sqrt{10/\beta_L^c}$ collection. The uniqueness of a critical temperature at each N_f , $T_c^{-1} =$

$N_t a(\beta_L^c) = N'_t a(\beta_L^{c'})$ with $N_t \neq N'_t$ gives a *thermal* step-scaling $\Delta\beta_L^c = \beta_L^c - \beta_L^{c'}$. A vanishing (decreasing) $\Delta\beta_L^c$ can be the signal of the (pre-)conformality.

Our thermal step-scaling is a function of N_f and two lattice temporal extensions, $\Delta\beta_L^c(N_f; N_t, N'_t)$, and tends to be smaller with increasing N_f . We here estimate the number of flavor satisfying $\Delta\beta_L^c(N_f^*) = 0$ by extrapolating our β_L^c collection into the larger flavor region. To this end, we plot the (pseudo) critical lattice coupling $g_L^c = \sqrt{10/\beta_L^c}$ as a function of N_f in Fig. 1, which gives an extension of Miransky-Yamawaki diagram [21] to finite T cases.

Let us first pick up the lattice critical couplings for $N_f = 6$ and 8, and consider a “constant N_t ” line. As shown in the left panel of Fig. 1, $N_t = 6$ and 12 lines get to a joint at $(g_L^c, N_f^*) = (1.825 \pm 0.02, 11.57 \pm 0.17)$, and $N_t = 6$ and 12 lines at $(g_L^c, N_f^*) = (1.753 \pm 0.02, 10.715 \pm 0.17)$, indicating the infra-red fixed point with vanishing thermal step scalings. Next we shall investigate the critical lattice couplings at $N_t = 6$ and 8 for whole range of $N_f = 0 - 8$. They can be well fitted by assuming the functional expression $N_f(g_L^c) = A \cdot \log [B \cdot (g_L^c - g_L^c|_{N_f=0}) + 1]$ giving $(g_L^c, N_f^*) = (1.88 \pm 0.09, 11.39 \pm 0.78)$ (right panel of Fig. 1). Thus, the thermal step-scaling with the use of our lattice critical couplings supports the emergence of conformal window near to 12 flavor system, whose (pre-)conformality is now under debate in the recent lattice studies [4].

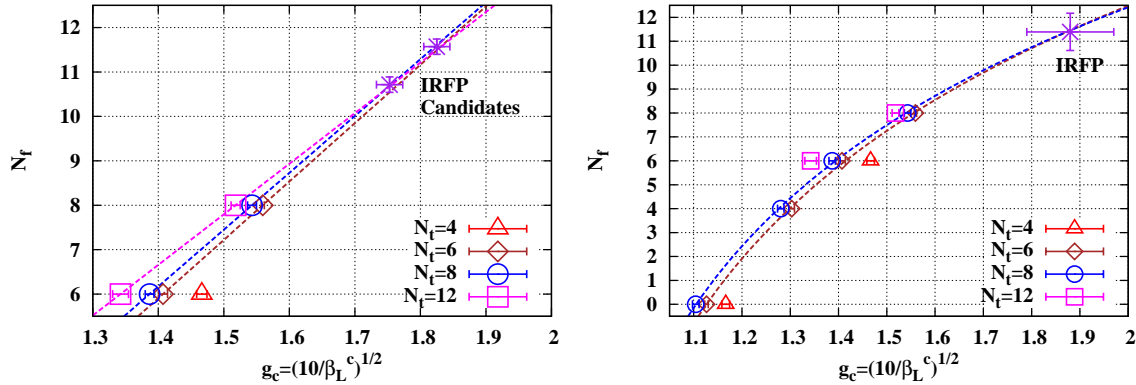


Figure 1. (Pseudo) critical values of the lattice coupling $g_L^c = \sqrt{10/\beta_L^c}$ for theories with $N_f = 0, 4, 6, 8$ and for several values of N_t in the Miransky-Yamawaki phase diagram. Left: We have picked up $g_L^c = \sqrt{10/\beta_L^c}$ for $N_f = 6$ and 8, and consider the “constant N_t ” line with $N_t = 6, 8, 12$. Candidates for the IRFP have been estimated by the crossing points of $N_t = 12$ line with $N_f = 6$ or $N_f = 8$ line. Right: The dashed line is a fit with the ansatz $N_f(g_L^c) = A \cdot \log [B \cdot (g_L^c - g_L^c|_{N_f=0}) + 1]$ for $N_t = 6$ and $N_t = 8$ results.

As indicated by the constant N_t line in Fig. 1, the critical coupling is an increasing function of N_f for a fixed lattice temporal extension. This behavior is the direct consequence of enhanced fermion screening effects due to the larger number of fermion species. We have observed the thermal step (or equivalently the asymptotic) scalings for our β_L^c , thereby, the enhancement of the screening effects gets to a physical significance. The test of the asymptotic scaling has been the historical homework since the pioneering finite T study at large N_f by J. Kogut and his collaborators [22] and now we have completed it.

We shall now investigate the N_f dependence of the critical temperature. In order to compare a physical quantity such as a critical temperature among different theories with a different N_f , it is necessary to introduce a N_f independent reference scale by hand. To set the reference scale, ideally speaking, we would like to measure various sizes of Wilson loops at various

numbers of flavors in Monte-Carlo simulations to obtain the running couplings \bar{g} in the wide range of scales at each number of flavor. In stead of performing such a massive simulation, we approximately construct the renormalization flow via the integral of the two-loop beta-function $\beta(g) = -g^3(b_0 + b_1g^2)$

$$\frac{T_c}{M(g_L^{\text{ref}})} = \frac{1}{N_t} \frac{a^{-1}(g_L^c)}{M(g_L^{\text{ref}})} = \frac{1}{N_t} \int_{g_L^{\text{ref}}}^{g_L^c} \frac{dg}{-g^3(b_0 + b_1g^2)}. \quad (3)$$

To specify the reference scale $M(g_L^{\text{ref}})$, we utilize our plaquette (tad-pole factor u_0) data shown in the left panel of Fig. 2. Note that the plaquettes can be regarded as a kind of renormalized couplings. Let us consider a constant u_0 without N_f dependences, for instance $u_0 = 0.9$ in figure, and read-off the corresponding bare lattice couplings at each N_f . The obtained $g_L(N_f) = \sqrt{10/\beta_L(N_f)}$ is used as a reference coupling g_L^{ref} in Eq. (3). This procedure imitates the scale setting in the potential scheme renormalization, and the use of N_f independent u_0 is motivated by the FRG scale setting method [12].

To be analogous to the FRG study, we should choose u_0 so as to get a UV $M(g_L)$ free from the chiral dynamics. The middle panel of Fig. 2 displays the N_f dependence of $T_c/M(g_L^{\text{ref}})$ defined by Eq. (3) with $u_0 = 0.9$. Fitting $T_c/M(g_L^{\text{ref}})$ with the FRG motivated ansatz $T_c = K|N_f^* - N_f|^{(-2b_0^2/b_1)(N_f^*)}$, we now read-off the lower edge of conformal window $N_f^* \sim 9.47 \pm 0.02$, which is somewhat smaller value comparing to those obtained by the vanishing thermal scale settings. In the middle panel of Fig. 2, we find $T_c/M(g_L^{\text{ref}}) \ll 1$, indicating the UV nature of the reference scale $M(g_L^{\text{ref}})$.

To get more transparent view for the UV reference scale, we here consider the particular reference coupling g_L^{ref} - the thermal critical coupling g_T^c which makes the reference scale $M(g_L^{\text{ref}} = g_T^c)$ be equivalent to T_c in Eq. (3): $T_c/M(g_T^c) = 1$, giving a typical interaction strength at T_c . As shown in the right panel of Fig. 2, the increasing nature of g_T^c indicates a realization of more strongly interacting non-Abelian plasma at larger N_f as discussed in Ref. [13]. The criterion to set the UV reference scale $M(g_L^{\text{ref}})$ at every N_f would be given by the condition $g_L^{\text{ref}} \ll g_T^c(N_f)$ for all N_f . We find that $u_0 \geq 0.84$ meets a requirement, while the use of too large u_0 suffers from the strong discretization errors. In practice, we find that the number of flavor giving the vanishing $T_c/M(g_L^{\text{ref}})$ is relatively stable within the range $0.84 \leq u_0 \leq 0.94$ which results in $9.85 \geq N_f^* \geq 9.17$.

4. Summary

We have investigated the (pre-)conformal dynamics in color SU(3) gauge theories with multi-species of fundamental fermions by using the lattice Monte-Carlo simulation. In order to study the conformality beyond the fixed number of flavor N_f , we have focused on a reducing chiral dynamics at finite T as a function of increasing N_f . We have observed stronger fermion screenings resulting from a larger fermion multiplicity at larger N_f . We have investigated a finite T step-scaling which follows from the uniqueness of critical temperature (T_c) at each N_f , then the vanishing step-scaling signals the conformal dynamics at $N_f^* \sim 10 - 12$. Further, motivated by the recent FRG based studies [12], we have examined the N_f dependence of T_c , by introducing a UV N_f independent reference scale $M(g_L^{\text{ref}})$. We have used the thermal critical coupling g_T^c as a criterion to insure the UV nature of $M(g_L^{\text{ref}})$ by imposing the condition $g_L^{\text{ref}} \ll g_T^c(N_f)$ for all N_f . We have found that the number of flavor giving a vanishing $T_c/M(g_L^{\text{ref}})$ is relatively stable within the range $0.84 \leq u_0 \leq 0.94$, which results in $9.85 \geq N_f^* \geq 9.17$.

As a future perspective, we should measure various sizes of Wilson loops at various numbers of flavors, and perform more rigorous scale settings in the potential scheme. It is also mandatory to investigate the chiral limit and the thermodynamic limit at large N_f . This, together with a

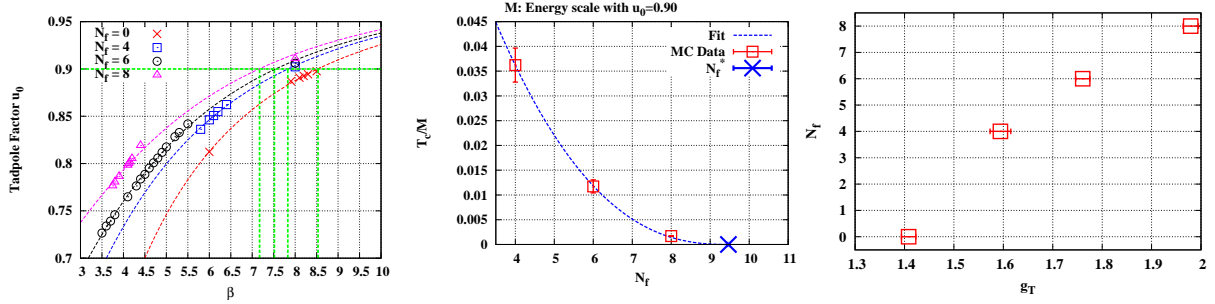


Figure 2. Left: The β_L dependences of the tadpole factor u_0 at zero temperature with the use of 12^4 lattice. Specifying a constant u_0 (e.g. $u_0 = 0.9$ in figure), we read off the corresponding lattice couplings β_L which are used to define the scale M at each theory with N_f . Middle: The N_f dependence of T_c/M where M is the UV scale with $u_0 = 0.9$ at each theory with N_f . The dashed line represents the fit for data by using the FRG motivated ansatz $T_c = K|N_f^* - N_f|^{(-2b_0^2/b_1)(N_f^*)}$ Right: The thermal critical coupling with increasing N_f .

more extended set of flavor numbers, will allow a quantitative analysis of the critical behavior in the vicinity of the conformal IR fixed point.

Acknowledgments

The author thanks Maria Paola Lombardo and Elisabetta Pallante for continuous discussions. He thanks Edward Shuryak for fruitful discussion during the xQCD workshop. This work was in part based on the MILC Collaboration's public lattice gauge theory code [15]. The numerical calculations were carried out on the IBM-SP6 and BG/P at CINECA, Italian-Grid-Infrastructures in Italy, and the Hitachi SR-16000 at YITP, Kyoto University in Japan.

References

- [1] W. E. Caswell, Phys. Rev. Lett. **33** (1974) 244.
- [2] T. Banks and A. Zaks, Nucl. Phys. B **196** (1982) 189.
- [3] For a recent review, see F. Sannino, Acta Phys. Polon. B **40** (2009) 3533.
- [4] For recent reviews, see L. Del Debbio, PoS **LATTICE2010** (2010) 004; E. Pallante, PoS **LATTICE2009** (2009) 015.
Phys. Rev. Lett. **100** (2008) 171607 [Erratum-ibid. **102** (2009) 149902].
- [5] A. Deuzeman, M. P. Lombardo and E. Pallante, Phys. Lett. B **670** (2008) 41.
- [6] T. Appelquist, G. T. Fleming and E. T. Neil, Phys. Rev. D **79** (2009) 076010.
- [7] A. Deuzeman, M. P. Lombardo and E. Pallante, Phys. Rev. D **82** (2010) 074503.
- [8] T. Appelquist, G. T. Fleming, M. F. Lin, E. T. Neil, D. A. Schaich, Phys. Rev. **D84**, 054501 (2011).
- [9] A. Hasenfratz, Phys. Rev. D **82** (2010) 014506.
- [10] Z. Fodor, K. Holland, J. Kuti, D. Nogradi and C. Schroeder, Phys. Lett. **B703** (2011) 348-358.
- [11] K. Miura, M. P. Lombardo and E. Pallante, Phys. Lett. B **710**, 676 (2012) [arXiv:1110.3152 [hep-lat]].
- [12] J. Braun, C. S. Fisher, H. Gies, Phys. Rev. **D84** (2011) 034045; J. Braun and H. Gies, JHEP **1005** (2010) 060; **0606** (2006) 024.
- [13] J. Liao, E. Shuryak and E. Shuryak, Phys. Rev. Lett. **109**, 152001 (2012) [arXiv:1206.3989 [hep-ph]].
- [14] For a recent development, see a review, U. Gursoy, E. Kiritsis, L. Mazzanti, G. Michalogiorgakis and F. Nitti, Lect. Notes Phys. **828**, 79 (2011) [arXiv:1006.5461 [hep-th]].
- [15] MILC Collaboration, <http://www.physics.indiana.edu/~sg/milc.html>
- [16] C. Bernard *et al.*, Phys. Rev. D **75** (2007) 094505; M. Luscher and P. Weisz, Phys. Lett. B **158** (1985) 250; Commun. Math. Phys. **97** (1985) 59 [Erratum-ibid. **98** (1985) 433].
- [17] G. P. Lepage, P. B. Mackenzie, Phys. Rev. D **48** (1993) 2250.
- [18] M. A. Clark, PoS **LAT2006** (2006) 004.
- [19] A. Kocic, J. B. Kogut and M. -P. Lombardo, Nucl. Phys. B **398** (1993) 376.
- [20] S. Gupta, Phys. Rev. D **64** (2001) 034507.

- [21] V. A. Miransky and K. Yamawaki, Phys. Rev. D **55** (1997) 5051 [Erratum-ibid. D **56** (1997) 3768].
- [22] J. B. Kogut, J. Polonyi, H. W. Wyld and D. K. Sinclair, Phys. Rev. Lett. **54**, 1475 (1985).